KUWAIT UNIVERSITY

Department of Mathematics and Computer Science

Math 102 | December 15, 2009
Calculus B | Second Midterm Exam | Time: 90 minutes

Use of calculators is not allowed in this exam. Please switch off your mobile phones

1. (2 points each) Evaluate the following limits:

(a) $\lim_{x\to 0} \dfrac{\cos(2x) - e^{3x^2}}{x^2}$ (b) $\lim_{x\to 1^+} (\ln x)^{\ln x}$

2. Evaluate the following integrals ($3\frac{1}{2}$ points each)

(a)

$$\int \sqrt[3]{\tan x}\ \sec^4 x\, dx$$

(b)

$$\int x^2 \sqrt{4 - x^2}\, dx$$

(c)

$$\int \frac{1}{(1 + e^x)^2}\, dx$$

(d)

$$\int \frac{10\cos x + 10}{2\cos x - \sin x + 2}\, dx$$

(e)

$$\int \frac{1}{\sqrt{x} + 2\sqrt[4]{x} + 2}\, dx$$

(f)

$$\int x^3 (\ln x)^2\, dx$$

Solution Key for Exam 2 Calculus B Fall 09

1) $\lim\limits_{x\to 0} \dfrac{\cos 2x - e^{3x^2}}{x^2} = \lim\limits_{x\to 0} \dfrac{-2\sin 2x - 6x e^{3x^2}}{2x} = \lim\limits_{x\to 0^+} \dfrac{-4\cos 2x - 6e^{3x^2} - 36x^2 e^{3x^2}}{2} = -5$

2) $\lim\limits_{x\to 1^+} \ln x\,(\ln(\ln x)) = \lim\limits_{x\to 1^+} \dfrac{\ln(\ln x)}{1/\ln x} = \lim\limits_{x\to 1^+} \dfrac{\frac{1}{\ln x}\,\frac{1}{x}}{\frac{-1}{(\ln x)^2}\,\frac{1}{x}} = \lim\limits_{x\to 1^+} -\ln x = 0$

$\Rightarrow \lim\limits_{x\to 1^+} (\ln x)^{\ln x} = e^0 = 1$

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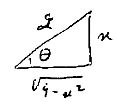
a) $u = \tan x$

$\int \tan^{1/3}x\,(1+\tan^2 x)\sec^2 x\,dx = \int u^{1/3} + u^{7/3}\,du = \frac{3}{4}(\tan x)^{4/3} + \frac{3}{10}(\tan x)^{10/3} + C$

b) $\int x^2\sqrt{4-x^2}\,dx = \int 4\sin^2\theta\,(2\cos\theta)(2\cos\theta)\,d\theta$ where $x = 2\sin\theta$



$= 16\int \sin^2\theta\cos^2\theta\,d\theta = 4\int \frac{1-\cos 4\theta}{2}\,d\theta$

$= 2\theta - \frac{\sin 4\theta}{2} + C = 2\theta - 2\sin\theta\cos\theta\,[\cos^2\theta - \sin^2\theta] + C$

$= 2\sin^{-1}\left(\frac{x}{2}\right) - \frac{x\sqrt{4-x^2}\,(2-x^2)}{4} + C$

c) $\int \frac{1}{(1+e^x)^2}\,dx$ $\quad u = 1+e^x$

$= \int \frac{1}{(u-1)u^2}\,du = \int \frac{-1}{u} - \frac{1}{u^2} + \frac{1}{u-1}\,du = -\ln|1+e^x| + \frac{1}{1+e^x} + x + C$.

d) $\int \frac{10\cos x + 10}{2\cos x - \sin x + 2}\,dx = \int \frac{10\,\frac{1-t^2}{1+t^2} - 10}{2\,\frac{1-t^2}{1+t^2} - \frac{2t}{1+t^2} + 2}\,\frac{dt}{1+t^2} = \int \frac{-10\,dt}{(t-2)(t^2+1)}$

$= \int \frac{2t+4}{t^2+1} + \frac{-2}{t-2}\,dt = \ln\left(1+\tan^2\frac{x}{2}\right) + 2x - 2\ln\left|\tan\frac{x}{2} - 2\right| + C$

e) $\int \frac{dx}{\sqrt{x} + 2\sqrt[4]{x} + 2}$ $\quad x = t^4 \quad dx = 4t^3\,dt$

$= \int \frac{4t^3\,dt}{t^2+2t+2} = \int 4t - 8 + \frac{8t+16}{t^2+2t+2}\,dt = 2\sqrt{x} - 8\sqrt[4]{x} + 4\ln(\sqrt{x} + 2\sqrt[4]{x} + 2) + 8\tan^{-1}(\sqrt[4]{x}+1) + C$

f) $\int x^3(\ln x)^2\,dx = \frac{x^4}{4}(\ln x)^2 - \frac{1}{2}\int x^3\ln x\,dx$

$= \frac{x^4}{4}(\ln x)^2 - \frac{1}{2}\left\{\frac{x^4}{4}\ln x - \int \frac{x^3}{4}\,dx\right\} = \frac{x^4(\ln x)^2}{4} - \frac{x^4\ln x}{8} - \frac{x^4}{32} + C$